INTERMEDIATE PERIOD RESPONSE OF WATER LEVELS IN WELLS TO CRUSTAL STRAIN:
SENSITIVITY AND NOISE LEVEL

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Abstract. The response of water levels in wells to earth tides indicates that wells can be used to detect small crustal strain. Vertical groundwater flow between the well intake and the water table can significantly attenuate this sensitivity. The attenuation of strain sensitivity as a function of frequency can be inferred from the response of water wells to atmospheric loading. For the wells examined in this study, significant attenuation due to water table drainage can occur when strain accumulates gradually over periods of days to weeks. Despite the presence of attenuation the wells are still sensitive strain meters over this range in period. At a frequency of 2.5 cycles/day, the noise level of the water level records examined is -130 dB relative to 1 strain^2/Hz. This noise level is about the same as that reported for dilatometers and wire strain meters, but is at least 10 dB higher than that reported for laser strain meters. In the frequency band of 0.025 to 2.5 cycles/day, the noise level of the water level records examined increases roughly 25 dB-per-decade decrease in frequency. Some of this noise is due to the influence of atmospheric loading. When the effects of atmospheric loading are removed from the record, the noise level is reduced to roughly 20 dB-per-decade for frequencies above 0.08 cycles/day, a rate typical of high-quality strain meters. For periods slightly less than a month, the wells have a lower noise level in terms of areal strain than that of the best geodetic distance measurements.

Introduction

Water level fluctuations in response to earth tides have long been observed in wells. Despite the apparent sensitivity of wells to strain and a well-established theory that quantitatively describes the interaction between pore pressure and elastic rock deformation [Biot, 1941] the use of water wells to monitor tectonic strain has been limited in extent [e.g., Sterling and Smets, 1971; Johnson et al., 1973, 1974; Roeloffs and Rudnicki, 1984; Lippincott et al., 1985]. The lack of use of water wells as strain meters is partly traceable to the often enigmatic coseismic response of water levels in wells to earthquakes [e.g., Bower and Heaton, 1978]. However, even if the coseismic response of water levels in wells could be generally explained, there would be some obvious problems with using water wells as strain meters. These problems can be separated into two broad classes: those that are related strictly to the bulk material properties of the rock near the well; those that are due primarily to groundwater flow.

The principal problem related to bulk material properties is that the sensitivity of a well to strain is highly dependent on the porosity and elastic properties of the rock or sediment that surround the well intake. A well open to rock or sediment that is highly compressible and porous cannot be expected to be highly sensitive to tectonic strain. This point is discussed in detail elsewhere (S. Rojstaczer and D. C. Agnew, The static response of the water level in an open well to areally extensive deformation under confined conditions, submitted to Journal of Geophysical Research, 1988, hereinafter referred to as submitted manuscript).

Many other problems with using water wells as strain meters are due primarily to the influence of groundwater flow. One problem related to groundwater flow is that the water level in a well responds to hydrology as well as to strain. For example, a year of above average rainfall will likely cause a long-term rise in water level. The influence of rainfall can be viewed as noise placed on any strain signal that might be present in the water level record.

Groundwater flow can cause another significant problem inherent in the use of water wells as strain meters; it may reduce strain sensitivity and thus restrict the usefulness of water wells to a narrow frequency band. There are at least three potential ways for groundwater flow to reduce strain sensitivity (see Figure 1). The first source is the limited hydraulic communication between the well and the saturated rock; if the well is to be a gage of pore pressure, changes in pore pressure must be accompanied by flow into or out of the borehole. For strains above some limiting frequency (that depends on the borehole geometry and the material properties of the saturated rock), groundwater flow will be insufficient to allow the full pore pressure signal to be seen in the well.
The second source of sensitivity reduction due to groundwater flow is large-scale horizontal transient flow from or to the monitored region of strain-induced pore pressure change. For strain of tectonic origin, the wavelengths of the pressure disturbance are likely of the order of kilometers or greater, so that this problem is probably significant only at very low frequencies.

The third source of sensitivity deterioration is vertical flow from or to the water table. Since the water table is particularly insensitive to crustal strain [Bredehoeft, 1967], any hydraulic connection between the water table and the zone of saturated rock monitored can cause significant attenuation of the strain-induced pore pressure signal. Water wells are typically in communication with rocks that are less than 200 m below the water table, and this source will likely be significant at frequencies higher than those of the second source.

The problem of noise in the water level signal has not been quantitatively examined, but problems of strain sensitivity related to groundwater flow have been studied previously in some detail. Sensitivity attenuation caused by fluid flow into and out of the well has been extensively examined [e.g., Cooper et al., 1965; Bodvarsson, 1970; Johnson, 1973; Gieske, 1986] and is discussed elsewhere [Rojstaczer, 1988]. The influence of the water table on attenuation has been examined by Johnson [1973], A. G. Johnson and A. Nur (unpublished manuscript, 1978), and Bower and Heaton [1978]; these studies theoretically examined this influence assuming that the unsaturated zone above the water table did not significantly influence water table response.

This study examines the influence of vertical groundwater flow to the water table on the sensitivity of water wells as strain meters and the noise levels of water wells in comparison with those of other strain meters. The influence of vertical groundwater flow on attenuation of sensitivity is examined by: extending the theoretical work of Johnson [1973] and A. G. Johnson and A. Nur (unpublished manuscript, 1978) to include the influence of the unsaturated zone above the water table on atmospheric pressure induced strain; applying these theoretical results to the response of two wells to atmospheric loading in order to examine how vertical groundwater flow influences strain sensitivity as a function of frequency. The comparative performance of water wells as strain meters is assessed by calibrating two wells to known strains and examining noise levels in the frequency band of 0.025 to 2.5 cycles/day.

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the pore space under drained conditions [Biot and Willis, 1957; Nur and Byars, 1971]:

$$\alpha = 1 - \frac{\beta_u}{\beta}$$  \hspace{1cm} (3)

It should be noted that $\beta_u$ and $\beta$ are the fluid and rock grain compressibility, respectively, and $\phi$ is the porosity.

In an open well, the static-confined response to atmospheric loading, $\sigma_a'$, is

$$E' = \frac{wp\beta}{\sigma_a} = 1 - \frac{2B(1+\nu)}{3(1-2\nu)\alpha B}$$  \hspace{1cm} (4)

The parameter, $E'$, is called the static-confined barometric efficiency (S. Rojstaczer and D. C. Agnew, submitted manuscript). Equation (4) differs slightly from the derivation of $E'$ that can be obtained from Van der Kamp and Gale [1983]. The essential difference is that (4) assumes that the atmospheric load is placed upon an elastic half-space. Under this assumption, atmospheric loading produces areal strain, $\epsilon_a'$, equal to the vertical strain, $\epsilon_v$, at typical well depths [Farrell, 1972]. Van der Kamp and Gale [1983] assume that surface loading induces vertical strain only (a traditional assumption in hydrology [e.g., Jacob, 1940]). The appropriateness of either assumption ($\epsilon = \epsilon_v$ or $\epsilon = 0$) is discussed elsewhere (S. Rojstaczer and D. C. Agnew, submitted manuscript).

The static-confined areal strain sensitivity, $A'$, and barometric efficiency, $E'$, influence the degree that groundwater flow is caused by areally extensive deformation. At typical well depths, groundwater flow in response to laterally extensive areal strain such as that imposed by earth tides and broad-scale tectonic deformation is governed by [Van der Kamp and Gale, 1983]

$$D' \frac{\partial^2 \rho}{\partial z^2} = \frac{\partial \rho}{\partial t} + \rho \beta \frac{\partial \epsilon}{\partial t} \frac{\partial}{\partial t}$$  \hspace{1cm} (5)

where $\rho$ is pore pressure, $D'$ is a hydraulic diffusivity for imposed areal strain under conditions of plane stress [Van der Kamp and Gale, 1983]:

$$D' = \frac{k \phi \rho \alpha \beta [1 - a(1-2\nu)]}{\mu \beta \beta_a' \phi(\beta \beta - \beta_u)}$$  \hspace{1cm} (6)

and $k$ and $\mu$ are the permeability of the formation and the viscosity of the pore fluid, respectively.

In response to areally extensive surface loading (such as that produced by atmospheric loading), groundwater flow at the near surface of a half-space is governed by (S. Rojstaczer and D. C. Agnew, submitted manuscript)

$$D \frac{\partial^2 \rho}{\partial z^2} = \frac{\partial \rho}{\partial t} + (1-E_B') \frac{\partial \alpha}{\partial t}$$  \hspace{1cm} (7)

where $D$ is a hydraulic diffusivity under conditions of areally extensive imposed surface stress:

$$D = \frac{k \rho \alpha \beta [1 - a(1-2\nu)]}{\mu \beta \beta_a' \phi(\beta \beta - \beta_u)}$$  \hspace{1cm} (8)

Equations (5) and (7) indicate that pore pressure response to imposed areal strain and atmospheric loading are both described by diffusion equations that contain a source term. The source term in the case of imposed areal strain has (owing to the hydraulic diffusivity, $D'$, being slightly larger than the hydraulic diffusivity, $D$, for equal-bulk material properties) a weaker influence on time dependent changes in pore pressure. The diffusion equations given above are an accurate description of pore pressure response as long as we can assume that in the frequency range of interest there is no lateral pore pressure dissipation.

Influence of Water Table Drainage on the Sensitivity of Water Levels in Wells to Areally Extensive Strain

Figure 2 shows idealized descriptions of water level response to two types of deformation: an imposed areal strain and an imposed atmospheric load. In both descriptions I neglect the influence of borehole drainage and assume that the water level in the well is a direct reflection of pore pressure in the rock.

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Fig. 2. (a) Idealized model of response of well to areally extensive periodic strain; (b) Idealized model of response of well to areally extensive periodic atmospheric loading.
formation. For the areal strain problem, the rock is subject to a periodic areal strain $A \cos(\omega t)$. The water table is at zero pressure and the well is an accurate gage of pore pressure.

For the atmospheric loading problem the rock is subject to a periodic vertical stress $A \cos(\omega t)$, and areal strain $\epsilon_A$ equals the vertical strain $\epsilon_V$. The water table is subject to a pressure $-A G \cos(\omega t + \delta)$, where $G$ and $\delta$ account for the attenuation and phase advance of the atmospheric pressure owing to diffusion of air through the unsaturated zone above the water table. The water in the well is subject to a periodic pressure $-A \cos(\omega t)$ (i.e., the well is open to the atmosphere), and the relation between water level, $w$, and pore pressure is $\frac{w - (A \cos(\omega t) + p)}{\rho g}$.

Since fluid flow is assumed to be one dimensional, both problems outlined above are readily solved analytically.

**Frequency Response of a Well to Imposed Areal Strain**

The response of a water well to imposed periodic areal strain is given by (Appendix A)

$$w = \frac{[AC \exp(-J') \cos(mt-J') - AC \cos(mt)]}{\rho g}$$

(10)

Where $A$ is the amplitude of the areal strain and $Q'$ is a dimensionless frequency:

$$Q' = \frac{z^2 \omega}{2D'}$$

(11)

The result given by (10) is qualitatively similar to the frequency dependent response of water wells to strain given by Johnson [1973] and A. G. Johnson and A. Nur (unpublished manuscript, 1978). The major differences between their analogous solution and that given here are due to their approximation of the water table as a spherically shaped boundary. Equation (11) indicates that the most important parameter governing sensitivity as a function of frequency is the depth from the water table, $z$. Hydraulic diffusivity is also an important factor, with low diffusivity favoring low attenuation at a given frequency.

The gain and phase (phase advance is positive) of the sensitivity to areal strain are plotted as a function of the dimensionless frequency, $Q'$, in Figure 3 under the assumption that the static-confined sensitivity of the well to areal strain, $A'$, is 0.05 cm water level drop per areal nanostrain. Attenuation and phase shift begin to significantly deviate from the static-confined response when the dimensionless frequency decreases to a value of 10. Between a dimensionless frequency of 1 and 10, the strain signal is slightly amplified relative to static-confined conditions; near-complete attenuation of the strain signal takes place for strain with dimensionless frequency 0.001 or less.

It is useful to examine the theoretical response of a well that taps rock with reasonable hydraulic and mechanical properties. Given a well tapping rock with a vertical permeability of
10^{-14} \text{m}^2 (the average permeability of nonargillaceous materials inferred for the crust \cite{Brace, 1980, 1984}), a matrix compressibility of 1 \times 10^{-10} \text{Pa}^{-1}, a fluid compressibility of 4 \times 10^{-10} \text{Pa}^{-1}, a solids compressibility of 2 \times 10^{-11} \text{Pa}^{-1}, a porosity of 0.10, and a well depth (relative to the water table) of 100 m, the dimensionless frequency will have a value greater than 10 for strain with frequency greater than 3 cycles/day. This result indicates that for reasonable material properties and geometries, vertical groundwater flow to the water table can have a strong influence on strain sensitivity in the frequency range of practical interest.

**Frequency Response of a Well to Atmospheric Loading**

The response of a well to atmospheric loading is considerably different than that given above. These differences are due primarily to the diffusion of the load through the air phase of the unsaturated zone \cite{Weeks, 1979}. Water level response in an open well to periodic fluctuations in atmospheric loading is given by (Appendix B)

\[ w = \left( -M + I \right) A \exp(-Q) \cos(\omega t - Q) - (\gamma - 1) A \cos(\omega t) - NA \exp(-Q) \sin(\omega t - Q) \right) / \rho g \]  

(12)

where \( M \) and \( N \) are

\[ M = \frac{2 \cosh(2\sqrt{\frac{Q}{R}})}{\cosh(2\sqrt{\frac{Q}{R}}) + \cos(2\sqrt{\frac{Q}{R}})} \]  

(13a)

\[ N = \frac{2 \sinh(\sqrt{\frac{Q}{R}})}{\cosh(2\sqrt{\frac{Q}{R}}) + \cos(2\sqrt{\frac{Q}{R}})} \]  

(13b)

Q and \( R \) are dimensionless frequencies referenced to fluid diffusivity, \( D \), and air diffusivity, \( D_a \), respectively:

\[ Q = \frac{z^2 \omega}{2D} \]  

(14a)

\[ R = \frac{L^2 \omega}{2D_a} \]  

(14b)

and \( L \) is the depth from the Earth's surface to the water table. The gain (barometric efficiency, \( E_b \)) and phase of the response of a water well to atmospheric pressure fluctuations are shown in Figure 4 for a well with a static-confined barometric efficiency, \( E_b^s \), of 0.5.

The response is plotted as a function of two dimensionless parameters: dimensionless frequency, \( Q \), and the ratio of dimensionless frequencies \( R/Q \) and \( Q \). The dimensionless ratio, \( R/Q \), is a measure of the time taken for atmospheric pressure changes to reach the water table versus the time taken for water table drainage to significantly influence water well response.

For values of \( R/Q \) less than 0.0001 the water table is fully influenced by atmospheric pressure changes at dimensionless frequency, \( Q \), and the response of the water well to atmospheric loading is functionally identical (except for a phase shift of 180°) to the response to strain which is applied directly to the solid phase alone (equation (10)). Under these conditions the attenuation of sensitivity to tectonic or earth tide induced strain can be inferred from the air pressure response once an allowance is made for the slightly different hydraulic diffusivity that governs fluid flow. For values of \( R/Q \) greater than 0.1 there is significant attenuation and phase shift of the atmospheric pressure signal at the water table; under these conditions we can
expect that over a frequency band whose width and
distance are a strong function of R/Q, the water
well response will be significantly amplified
relative to the static-confined response. When
R/Q is greater than 10, we can expect a water
well to respond significantly to atmospheric
loading in a frequency band where response to
tectonic or earth tide induced strain is strongly
attenuated. Hence the response of a water well
to atmospheric loading, when R/Q is significantly
greater than 0, is not indicative of how the
sensitivity to tidal or tectonic strain
attenuates with frequency.

Although it is not possible, on the basis of
the response to atmospheric loading, to directly
infer how the sensitivity of water wells will
attenuate in response to imposed areal strain
when R/Q is 0.1 or greater, the atmospheric
pressure response gives us an indirect means to
determine how water levels in wells respond to
tectonic strain. The material and fluid flow
properties of the rock and/or sediment which
control well response can be determined by
fitting the atmospheric pressure response of a
well to the theoretical response (equation (12)
or (B6)). These elastic and fluid flow
properties can then be used in conjunction with
the theoretical response to imposed areal strain
(equation (10) or (A3)) to infer how sensitivity
to tectonic strain attenuates with frequency.

Overview of Wells to Be Examined

Figure 5 shows the water level record for the
two wells examined here. These wells were chosen
because: they are located in areas which have
been the focus of many crustal deformation
studies over the past decade; and they exhibited
the largest amplitude response to the strain
induced by earth tides of any of the wells
monitored in these areas over the time shown.
The hydrograph labeled LKT is the water level
record from a well in the Long Valley caldera
near Mammoth Lakes, California; the hydrograph
labeled TF is the record from a well near the San
Andreas fault at Parkfield, California.

The water level records at LKT and TF over the
second half of 1985 both show a long-term decline
in water level; this low-frequency decline is at
least partially due to the relative lack of
precipitation in central California over the
winter of 1984-1985. The power spectra of the
water level records are shown in Figure 6. Both
wells have considerable power in the semidiurnal
and diurnal frequency bands owing to earth tides
and atmospheric loading. Power levels increase
with decreasing frequency.

Figure 6 also shows the power spectra of
atmospheric pressure at TF and LKT over the
second half of 1985 (pressure is in terms of
equivalent water level). Comparison of the
atmospheric pressure power spectra with the water
level power spectra at TF and LKT indicates many
similarities. There is substantial power in the
semidiurnal and diurnal band and a strong
increase in power with decreasing frequency.

The depth and near-hole lateral permeabilities
of these two wells are shown in Table 1. For
both wells, the well intake is isolated from the
near surface to a depth in excess of 100 m. The
depth to water is about 20 m at both LKT and TF;
if near-hydrostatic conditions prevail at these
wells, then this depth indicates the depth to the
water table at both sites. The lateral
permeabilities of the rock in direct hydraulic
communication with these wells were inferred from
a slug test [Kipp, 1985] and pumpage data from
LKT and TF, respectively. These permeabilities
are high in relation to the range of
permeabilities that have been measured or
inferred for the crust [Brace, 1980, 1984]
indicating that hydraulic communication between
the well and rock is relatively good. The
borehole diameter is roughly 15 cm at both wells.

At each site, atmospheric pressure and water
level were measured with silicon strain bridge
transducers four times per hour. The water level
and atmospheric pressure records over the time
interval shown in Figure 5 contain 2% and 5% gaps
at LKT and TF, respectively; these gaps range
from 1/2 hour to a few days in length. The
method by which these gaps in the time series
were filled is described in Appendix C. The
tidal areal strain time series at each site was
determined from the body tide with no corrections
made for ocean loading, topographic, or geologic
effects. The results of Beaumont and Berger
[1975], Berger and Beaumont [1976] and Evans et
al. [1979] suggest that the amplitude of actual
tidal strain agrees with those determined from
the homogeneous earth tide with an error of about
±50%.

Inferred Strain Sensitivity of Wells

The relation of water level to crustal strain
induced by atmospheric loading and earth tides at
the two wells was determined from cross-spectral
estimation (Appendix D). Table 2 indicates the response of these wells to the $M_2$ and $O_1$ earth tides in terms of their areal strain sensitivity (centimeters/nanostrain) and phase. Their response to the $M_2$ tide in terms of areal strain sensitivity is identical; for both sites the areal strain sensitivity with regard to the $M_2$ component of the tidal potential is 0.034 cm/nanostrain. The areal strain sensitivity of the $O_1$ tide is slightly lower, particularly at TF. At TF this slightly greater difference in amplitude may be due partly to water table drainage which (as is shown below) significantly attenuates strain sensitivity in the frequency band of interest (0.025 to 2.5 cycles/day). Phase differences at LKT and TF between the $O_1$ and $M_2$ tidal constituents are slight. The phase shift of both constituents as well as the small difference in areal strain sensitivity between the $M_2$ and $O_1$ constituents at LKT are likely due to elastic anisotropy, local inhomogeneities, or ocean loading effects. At TF the phase shift of both constituents may be due to a combination of water table drainage, elastic inhomogeneity or anisotropy, and ocean loading influences.

The response of LKT to atmospheric loading is shown in Figure 7. The phase is generally flat and near 0° for frequencies greater than 0.02
TABLE 1. Description of Wells Examined

<table>
<thead>
<tr>
<th>Well Name</th>
<th>Location</th>
<th>Open Interval, m</th>
<th>Local Permeability, ( \text{m}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LKT</td>
<td>Long Valley</td>
<td>152-296</td>
<td>2 ( \times 10^{-11} )</td>
</tr>
<tr>
<td>TF</td>
<td>Parkfield</td>
<td>152-177</td>
<td>2 ( \times 10^{-14} )</td>
</tr>
</tbody>
</table>

Local permeability refers to the permeability near the borehole.

cycles/day. The admittance or barometric efficiency begins to show some attenuation below a frequency of 0.05 cycles/day. This attenuation may not reflect any influence of groundwater flow to the water table; rather it may be the result of some error in the estimate of barometric efficiency at low frequencies. For this study, use of error estimates for the admittance determined from cross-spectral estimation [e.g., Bendat and Piersol, 1986] yield the following 95% confidence interval for the barometric efficiency, \( E_{\beta,95} \):

\[
E_{\beta,95} = E_{\beta} \pm 1.2(1-F^2)E_{\beta} \tag{15}
\]

where \( F \) is the coherence. Figure 8 shows the coherence squared, as a function of frequency for LKT. At frequencies less than 0.05 cycles/day, the coherence squared is substantially less than 1, and the error bounds on barometric efficiency are greater than \( \pm 0.27 \). A flat barometric efficiency of 0.48 (the mean value of the barometric efficiency for frequencies greater than 0.05 cycles/day) over the entire frequency range examined is thus not inconsistent with the admittance values and is consistent with the general frequency insensitivity of the phase. Alternatively, the attenuation that begins to occur at frequencies less than 0.05 cycles/day is a real physical phenomenon, presumably due to the influence of groundwater flow to the water table.

The fit to the data, based on the assumption that there is some water table drainage in the observed frequency band, is also shown in Figure 7. The best fit to the data is achieved with a static-confined barometric efficiency, \( E_{\beta} \), of 0.45, a value for dimensionless frequency \( Q \) of 4.8, and a value for dimensionless frequency \( R \) of 0.94, where frequency \( \omega \) is in units of radians/day. This value of \( R \) yields a pneumatic diffusivity of \( 2 \times 10^{-3} \text{ m}^2/\text{s} \). Since pneumatic diffusivities under field conditions are not commonly measured, it is difficult to assess whether or not the pneumatic diffusivity inferred here is reasonable. It is a factor of 30 lower than a pneumatic diffusivity inferred by Weeks.

TABLE 2. Tidal Response of Wells Examined

<table>
<thead>
<tr>
<th>Well Name</th>
<th>( O_1 ) Gain</th>
<th>( O_1 ) Phase</th>
<th>( M_2 ) Gain</th>
<th>( M_2 ) Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>LKT</td>
<td>0.031</td>
<td>-166</td>
<td>0.034</td>
<td>-171</td>
</tr>
<tr>
<td>TF</td>
<td>0.030</td>
<td>-189</td>
<td>0.034</td>
<td>-188</td>
</tr>
</tbody>
</table>

Gain expressed in terms of centimeters of water level drop per areal nanostrain. Phase expressed in degrees.
Fig. 8. Multiple coherence squared for relation of water level to atmospheric loading and earth tides at LKT and TF.

[1979] from the response of a well in Lubbock, Texas, to atmospheric loading.

The value of $Q'$ consistent with the atmospheric loading response is $4.1\omega$. This value for $Q'$ indicates that the observed $M_2$ and $O_1$ tidal responses are not influenced by water table drainage. The value for the static-confined barometric efficiency given by this fit combined with the static-confined areal strain sensitivity estimated from the $M_2$ tidal response (shown in Table 2) allow us to calculate (S. Rojstaczer and D. C. Agnew, submitted manuscript) a compressibility for the rock in communication with the well. Assuming a Poisson's ratio of 0.25, the drained rock matrix compressibility is $1.5 \times 10^{-10}$ Pa$^{-1}$. Given this compressibility and assuming that the depth to water is indicative of the depth to the water table, it is also possible to estimate the vertical permeability of the zone between the water table and the depth of the well intake. This permeability is $7 \times 10^{-18}$ m$^2$, which is much less than the lateral permeability given in Table 1. The difference indicates either that moderately low permeability layers exist above the monitored zone or that there is considerable anisotropy in the permeability of the rock and sediment above the zone monitored. It should be noted that this inferred permeability is a maximum permeability based upon the assumption that the attenuation indicated at low frequencies is a real phenomenon.

The elastic and fluid flow properties determined from the response of LKT to atmospheric loading are used to infer how this well responds to crustal strain (Figure 9; note that we assume that the attenuated response to atmospheric loading reflects a real physical phenomenon). In terms of attenuation of sensitivity, it is not appreciably different than the atmospheric loading response. This correspondence suggests that the water table is in good pneumatic communication with the Earth's surface. The inferred sensitivity remains moderately high when frequencies are less than 0.001 cycles/day.

The response of TF to atmospheric loading is shown in Figure 10. This response has a signature which indicates that the influence of groundwater flow to the water table is present throughout the observed frequency band. The

Fig. 9. Inferred strain response of LKT and TF in terms of (a) areal strain sensitivity and (b) phase.
constituents shown in Table 2 and is also inconsistent with the phase of the atmospheric loading response. Rather, this decreasing sensitivity at higher frequencies is likely due to air pressure diffusion effects which amplify the atmospheric loading response in the frequency band of 0.2 to 2 cycles/day. As Figure 4 indicates, such a response would be theoretically possible if the dimensionless numbers, R and Q, were of the same magnitude.

The best fit to the data is given with a static-confined barometric efficiency of 0.37 and a value for both Q and R of 0.34, where frequency is in terms of radians/day. The fit to the atmospheric loading response indicates that water table drainage also influences areal strain response at frequencies less than about 5 cycles/day. The value of R yields a pneumatic diffusivity of $5 \times 10^{-3}$ m$^2$/s. The value for Q, which is consistent with the atmospheric loading response is 0.27. Fitting the strain response to the observed $M_2$ areal strain sensitivity given in Table 2 yields a static-confined areal strain sensitivity of 0.033 cm/n. These values of static-confined barometric efficiency and areal strain sensitivity indicate that the drained rock matrix compressibility for the rock in direct communication with the well bore is $1.9 \times 10^{-19}$ Pa$^{-1}$ (S. Rojstaczer and D. C. Agnew, submitted manuscript). Given this compressibility, the vertical permeability above the well intake is estimated to be $1 \times 10^{-14}$ m$^2$, a value that is only slightly less than the lateral permeability inferred from pumping data. This correspondence indicates that the material above and within the zone monitored is largely homogeneous and isotropic with respect to permeability.

The inferred response to tectonic strain is shown in Figure 9. In terms of attenuation of sensitivity it is considerably different than the atmospheric loading response. The inferred response to strain approaches static-confined conditions at a frequency of 2.5 cycles/day, but the static-confined response is never fully observed in the frequency band analyzed. Below a frequency of 0.001 cycles/day, where strain sensitivity is less than 1/10 of the inferred static-confined response, strain sensitivity asymptotically approaches zero.

**Strain Spectra of Wells**

The raw and filtered (atmospheric loading effects removed) strain spectra for LKT and TF are shown in Figure 11. These strain spectra were derived by (1) obtaining power spectral densities for the water level records in terms of centimeters$^2$/per cycle/day; (2) converting these spectra into units of strain$^2$/hertz through the use of the estimated static-confined areal strain sensitivities for the wells (0.034 cm/n for LKT and 0.033 cm/n for TF); and (3) adjusting the strain
Fig. 11. Power spectral densities at (a) LKT and (b) TF normalized to 1 strain²/Hz. "RAW" curve is water level record without atmospheric loading effects removed. "FILTERED" curve is water level record with atmospheric loading effects removed with a frequency dependent transfer function. "SIMFILT" curve is water level record with atmospheric loading effects removed with a filter of length one.

spectra, when necessary, to account for any unambiguous frequency dependent changes in areal strain sensitivity inferred from the response to atmospheric loading; (4) normalizing the spectra relative to 1 strain²/Hz and converting to a decibel scale. The strain spectra were determined with no detrending of the data.

Analysis of the computation of the strain spectra indicated that including the trends caused only slight changes in the strain spectra (generally less than 1 dB) for frequencies greater than 0.05 cycles/day.

The spectral estimates TF were adjusted upward to account for the observed decreasing sensitivity with period by essentially adding a slope of 8 dB-per-decade decrease in frequency to the strain spectra for frequencies less than 1 cycle/day; since the observed attenuation at LKT is slight and ambiguous in the frequency band examined here, no adjustments in the strain spectra were made. The raw strain spectra at LKT and TF are very similar. At 2.5 cycles/day the noise levels for both wells are at about -130 dB relative to 1 strain²/Hz. This noise level is at least 10 dB higher than that reported for laser strain meters (Berger and Levine, 1974; Beavan and Goulty, 1977; Agnew, 1986) but is comparable to that reported for dilatometers (Johnston et al., 1986) and wire strain meters (Bilham et al., 1974). At higher frequencies, noise levels increase at a rate of roughly 25 dB-per-decade decrease in frequency, a rate that is slightly high compared to at least one dilatometer (Johnston et al., 1986). The M₄ tide at both wells is roughly 20 dB above the background noise level.

Considerable reduction in noise level can be achieved by removing the effects of atmospheric loading on water level response. These effects were removed in two ways. In the first method the fit of the observed frequency response shown in Figures 7 and 10 was assumed to be the transfer function between water level and atmospheric pressure. This transfer function was then multiplied with the Fourier transform of the atmospheric pressure record, and the resultant frequency response was then inverted into the time domain and subtracted from the water level time series. In the second method a single coefficient between the water level record and the atmospheric pressure record was found by linear regression of the two time series (the coefficient was 0.48 for LKT and 0.35 for TF); the atmospheric pressure record was then simply multiplied by this coefficient, and the resultant time series was then subtracted from the water level record.

At LKT the strain spectra for the time series obtained with regression and the frequency dependent transfer function are very similar in the frequency range of interest. In the frequency band of 0.05 to 0.8 cycles/day, both strain spectra have noise levels as much as 7 dB lower than that obtained from the unfiltered water level record. The regression filter is slightly better at lowering the noise level of the water level record, suggesting that the inferred frequency dependent transfer function overestimates the effects of water table drainage on water level response to atmospheric loading.

At TF there is substantial difference between the strain spectra determined by filtering with the regression coefficient and the frequency
dependent transfer function. The difference is due to the considerable attenuation and phase shift of the atmospheric pressure signal in the water level record. Use of the frequency dependent transfer function reduces noise levels as much as 12 dB in the frequency band of 0.05 to 0.8 cycles/day. The filtered strain spectra do not have a significantly lower noise level than the unfiltered strain spectrum for frequencies greater than 0.9 cycles/day.

Both filtered records at LKT and the record filtered with a frequency dependent transfer function at TF yield strain spectra whose slopes increase at a rate of roughly 20 dB-per-decade in frequency in the frequency band of 0.08 to 2.5 cycles per day. This is comparable to the rate of increase seen in this frequency band for the strain spectrum reported for a dilatometer [Johnston et al., 1986]. Presumably, one could also remove the effects of atmospheric loading on dilatometers [Suyehiro, 1982] and significantly reduce the noise level of the instrumentation. For instruments that measure horizontal strain only, however, the reduction in noise level achieved by removing atmospheric loading effects may be slight [Bilham et al., 1974]. The similarity between the noise levels in these wells and those of dilatometers and wire strain meters suggests that either the effects of precipitation on the water level record are small for frequencies greater than 0.08 cycles/day or are similar to those of other strain meters.

Following the approach of Agnew [1987], I compare the strain response shown in Figure 11 with standard electronic distance measurement response by transforming standard error estimates for distance measurements into strain spectra. The most accurate electronic distance measurement currently available is obtained with a two-color laser device [Linker et al., 1986]. The standard error for a two-color distance measurement taken over a baseline of 10 km is 1.2 x 10^-7. If distance measurements 10 km in length were made to determine areal strain with a two-color geodimeter once a day, the resultant strain spectrum would be (in the absence of a tectonic signal) a flat -83 dB relative to 1 strain^2/Hz. The raw and corrected strain spectra for both wells are well below this noise level until frequencies are less than 0.04 cycles/day. This comparison indicates that these wells outperform the ability of electronic distance measurements to measure areal strain for deformation that takes place over days to weeks.

Conclusions

The results given here suggest that theoretical models which describe the response of water wells to atmospheric loading and areal strain can be used, in conjunction with cross-spectral estimation, to yield some valuable information on the influence that vertical groundwater flow to the water table has on strain sensitivity. When the water table is weakly isolated from atmospheric pressure changes, the attenuation of the atmospheric loading response of water wells is qualitatively similar to the attenuation of the earth tide and tectonic strain response. Well sensitivity in both cases gradually attenuates with decreasing frequency owing to water table drainage, and the frequency at which significant attenuation begins to take place is a strong function of well geometry and rock material properties. For wells open to rock in excess of 100 m below the water table, attenuation of sensitivity will be slight at periods of days to weeks if the rock above the open interval has a vertical permeability of 1 x 10^-14 m^2 or less and a compressibility of 1 x 10^-10 Pa^-1.

The wells examined here are most useful as strain meters over a limited frequency band. Both have a lower noise level than standard distance measurement techniques for strain events with periods of days to weeks; at longer periods, electronic distance measurements can have superior performance. At TF, increasing noise with increasing period appears to be the result of decreasing strain sensitivity in the presence of a 15 to 20 dB-per-decade increase in the water level power spectrum. At LKT, the increasing noise appears to be largely due simply to a 20 to 25 dB-per-decade increase in the water level spectrum.

The noise levels of these wells in the frequency band of 0.08 to 2.5 cycles/day compare well with dilatometers and wire strain meters. Laser strain meters offer comparatively better performance. While water wells may be limited in their ability to accurately detect teleseismic strain, the relative quality of these wells as strain meters in the frequency band analyzed here suggests that wells can be of considerable value in continuously monitoring intermediate period strain. In areas where conventional means of continuously monitoring strain may be difficult to install or maintain, water wells may provide a unique way of monitoring deformation. In areas where strain monitoring networks exist, it is possible that water wells can augment the ability of the network to reliably detect aseismic crustal deformation.

Appendix A: Solution to the Response of a Well to Periodic Areal Strain

The solution to the response of a well to periodic areal strain is obtained by solving for the pore pressure response to periodic areal strain. I assume that the well is an accurate gage of the average pore pressure of the saturated rock with which it is in hydraulic communication and also assume that the open interval of the well is very small relative to the change in pore pressure with depth. Pore pressure response to a periodic areal strain, A cos(\omega t), is governed by the equation:

\[ \frac{dP}{dt} = \frac{K}{\rho g} \frac{dS}{dx} \]

where \( P \) is the pore pressure, \( \rho \) is the density of the fluid, \( g \) is the acceleration due to gravity, \( K \) is the hydraulic conductivity, and \( S \) is the areal strain. The solution to this equation is given by:

\[ P(x,t) = \frac{1}{2} P_0 + \frac{1}{2} P_{max} \cos(kx - \omega t) \]

where \( P_0 \) and \( P_{max} \) are the initial and maximum pore pressures, \( k \) is the wave number, and \( \omega \) is the angular frequency of the strain. The wave number and angular frequency are related by the wave speed in the rock, \( c = \sqrt{K/\rho g} \).

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Equation (A1) was obtained from (5) by substituting $A \cos(\omega t)$ for $\epsilon$. This equation must be solved subject to the boundary conditions:

$$p(0,t) = 0 \quad (A2a)$$

$$p(\infty, t) = -\rho g A_s A \cos(\omega t) \quad (A2b)$$

where $z=0$ is taken to be the water table. No initial condition is imposed because we seek the periodic steady state solution. The solution in terms of gain and phase is

$$\text{Gain} = \left| \frac{P}{\rho g A_s A} \right| = \sqrt{(J^2+K^2)} \quad (A3a)$$

$$\text{Phase} = \tan^{-1}(-K/J) \quad (A3b)$$

where $J$ and $K$ are

$$J = \exp(-Q') \cos(\gamma Q') - 1 \quad (A4a)$$

$$K = \exp(-Q') \sin(\gamma Q') \quad (A4b)$$

Since the well is assumed to be an accurate gauge of pore pressure in the frequency band of interest, water level changes are related to pore pressure changes by $w=p/g$, and the solution in terms of water level change per change in strain can be obtained from (A3) by multiplying the gain by $A_s^2$. The solution in the real domain is given in (10).

**Appendix B: Solution to the Response of a Well to Periodic Atmospheric Loading**

The solution for the response of a well to periodic atmospheric loading is obtained, as in Appendix A, by solving for the pore pressure response. In Appendix A it was assumed that the water table ($z=0$) was always at zero pressure; this is not the case for periodic atmospheric loading, and before pressure at depth can be known, the water table pressure must be determined. The water table response to periodic atmospheric loading is determined through the use of a diffusion equation for flow of air through unsaturated porous materials [Buckingham, 1904; Weeks, 1979]:

$$D \frac{\partial^2 p_a}{\partial z^2} = \frac{\partial p_a}{\partial t} \quad (B1)$$

subject to the following boundary conditions:

$$p_a(-T,t) = A \cos(\omega t) \quad (B2a)$$

$$p_a(T,t) = -A \cos(\omega t) \quad (B2b)$$

where $p_a$ is the air pressure. The boundary $-T$ is taken to be the Earth's surface; the zone from depth 0 to depth $T$ is simply an artifice to assure that at the water table, $z=0$, there is no air flux. As in Appendix A, we seek the periodic steady state solution. The air pressure at the water table ($z=0$) is readily solved by employing complex notation:

$$p_a = MA \cos(\omega t) + NA \sin(\omega t) \quad (B3)$$

where $M$ and $N$ are given in equation 13. The pore pressure at the water table is $p_a^w$.

With the pressure at the water table known, the solution to pore pressure response to a periodic atmospheric pressure fluctuation, $A \cos(\omega t)$, is obtained from the following modified version of (7):

$$D \frac{\partial^2 p_a}{\partial z^2} = \frac{\partial p_a}{\partial t} - (1-E') A \cos(\omega t) \quad (B4)$$

The appropriate boundary conditions are

$$p_a(0,t) = -MA \cos(\omega t) - NA \sin(\omega t) \quad (B5a)$$

$$p_a(-T,t) = -A \cos(\omega t) \quad (B5b)$$

The solution in terms of gain and phase is given by

$$\text{Gain} = \left| \frac{\omega pg}{A} \right| = \sqrt{(U^2+V^2)} \quad (B6a)$$

$$\text{Phase} = \tan^{-1}(-V/U) \quad (B6b)$$

where $U$ and $V$ are

$$U = (-M+7) \exp(-\gamma Q) \cos(\gamma Q) + N \exp(-Q) \sin(\gamma Q) - (\gamma - 1) \quad (B7a)$$

$$V = (-M+7) \exp(-\gamma Q) \sin(\gamma Q) - N \exp(-Q) \cos(\gamma Q) \quad (B7b)$$

The response in terms of water level within the well can be obtained (as in (9)) by adding $A \cos(\omega t)$ to the real part of the solution and dividing by $pg$. The solution in the real domain in terms of water level is given in (12).

**Appendix C: Method by Which Gaps Were Filled in the Water Level and Atmospheric Pressure Record**

Gaps in the water level and atmospheric pressure record over the time period examined were filled by an iterative process. The gaps were originally filled by linear interpolation. A zero-phase high-pass filter [Otnes and Enochson, 1978] with a cutoff frequency of $10^{-8}$ Hz was then applied to the water level and atmospheric pressure data to remove any long-term trends. The autocovariance with a maximum length of 40 days was then calculated for each time series. Gaps were then filled with a symmetric linear filter of length 120 with weights determined from the structure of the
autocovariance and the distance between the interpolated point and the nearest data point. The spacing between each value used in the filter was eight data points (2 hours), and weights were calculated by solving the following system of linear equations:

\[
\begin{bmatrix}
\sigma^2 & C_{i+j}^1 & \cdots & C_{i+j}^{6n} & 1 & w_i & C_i^1 \cdots C_i^{6n} \\
C_{i+j}^1 & \sigma^2 & \cdots & C_{i+j}^{6n} & 1 & w_i & C^1 \cdots C^{6n} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
C_{i+j}^{6n+1} & \cdots & \sigma^2 & 1 & w_i & C_{i+j}^{6n+1} \cdots C_{i+j}^{6n+6} \\
1 & 1 & 1 & 1 & 0 & \mu & 1
\end{bmatrix}
\]

where \( \sigma^2 \) is the variance, \( \mu \) is the mean, \( C_{i+j} \) is the autocovariance between the \( i \)th data value and the \( j \)th data value used in the interpolation, \( w_i \) is the weight of the \( i \)th data point in the filter, and \( C_{i+j}^1 \) denotes the autocovariance for the length between the interpolated point and the \( i \)th nearest data value. This linear system of equations provides an estimate of the interpolated value for which the estimation variance is at a minimum [Journel and Huijbregts, 1978]. The estimation variance \( \sigma^2 \) is defined as

\[
\sigma^2 = \sigma^2 + \mu - \sum w_i C_{i+j}^1
\]

Once the gaps were interpolated using the system of equations given in (C1), the autocovariance was recalculated, and the system of equations was solved again. This process was repeated until convergence was achieved. The residual time series (the original time series with linearly interpolated gaps minus the high pass filtered time series with linearly interpolated gaps) was then added to the new time series to preserve the long-term trend.

Appendix D: Method by Which the Transfer Functions of Water Level to Earth Tides and Atmospheric Pressure Were Determined

The relation of water level to earth tides and atmospheric pressure at the wells in the frequency domain was obtained by solving the following complex system of equations for every frequency [e.g., Bendat and Piersol, 1986]:

\[
\begin{bmatrix}
BB & BT^- & HB^- \\
TB & TT^- & HT^-
\end{bmatrix}
= \begin{bmatrix}
BB^- \\
TB^- \\
TW^-
\end{bmatrix}
\]

(D1)

where \( BB \) and \( TT \) denote the power spectra of the atmospheric pressure and earth tides, respectively; \( BT \) denotes the cross spectrum between atmospheric pressure and earth tides; \( TB \) denotes the complex conjugate of the cross spectrum between atmospheric pressure and earth tides; \( BW \) and \( TW \) denote the cross spectra between atmospheric pressure and water level and earth tides, respectively; and \( HB \) and \( HT \) denote the transfer function between water level and atmospheric pressure and water level and earth tides, respectively.

The power spectra and cross spectra were obtained for the time series of interest by using the Blackman-Tukey procedure after removing the mean and long-term trend from the time series. This procedure is computationally inefficient because it requires that the autocorrelation or cross-correlation function be calculated in the real domain in order to obtain spectral quantities; it was used because the frequencies of interest were low relative to the length of the data set. The spectral estimates obtained from the Blackman-Tukey procedure were smoothed using a Hanning window.

At TF the long-term trend was removed with a zero-phase high-pass filter [Otnes and Enochson, 1978] with a cutoff frequency of 10^{-8} Hz. Spectral estimates were then obtained using a maximum correlation length of 20 days. At LKT the time series for water level, atmospheric pressure, and earth tides were lengthened to include the first half of 1986, and the data were decimated to two samples per hour; the time series was lengthened to see whether any attenuation of sensitivity could be unambiguously identified at low frequencies. The long-term trend was removed with a zero-phase high-pass filter with a cutoff frequency of 5 x 10^{-9} Hz. Spectral estimates were obtained using a maximum correlation length of 40 days.

The barometric and areal strain sensitivity as a function of frequency are simply the admittance of the transfer functions \( HB \) and \( HT \), respectively. The multiple coherence squared, \( \Gamma^2 \), of the transfer functions as a function of frequency was obtained from the following spectral estimates [e.g., Otnes and Enochson, 1978]:

\[
\Gamma^2 = \frac{(WB \times TT - WT \times TB)BW + (WT \times BB - WB \times BT)TW}{(TT \times BB - TB \times BT)WW}
\]

(D2)

where \( WB \) and \( WT \) denote the complex conjugates of \( BW \) and \( TW \), respectively, and \( WW \) is the water level power spectrum.

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